Distributed and fault tolerant control for a class of discrete time linear systems

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I. Extended Abstract

Teams of networked robots, i.e. multiple autonomous robots connected one to the others via a communication network, have been object of widespread research in the last years mainly because of their flexibility and robustness. A relevant part of the research in the field deals with the distributed control of such systems [1], since a decentralized control strategy is often a desirable feature and, sometime, the only one possible; this might be due to limited computation power and/or limited communication bandwidth, that make impossible the communication between a central unit and all agents in the team.

With regards to common problems in the framework of multi-agent systems, several authors have proposed solutions to consensus as in [2], [3]. Furthermore, in [4] the authors propose a partially decentralized algorithm to control the network centroid, variance, and orientation of the system. Some of the authors of this paper devised in [5] an observer-controller scheme for the distributed tracking of time-varying global task variables.

In this abstract, the authors describe a solution to the fault tolerant control of distributed multi-robot systems that builds on results in [5]. In fact, the robustness to faults of multi-agent systems is only potential and the failure of one or more units might jeopardize the all mission. The lack of a central unit and, then, of information about the overall system makes difficult the task of detecting and identifying faulty units. Therefore, although several FDI approaches have been presented in the last decades for single unit systems, both for continuous [6] and discrete-time systems [7], only few works dealt with the case of decentralized multi-agents systems.

For example, observer-based FDI approaches have been proposed for distributed systems as in [8] and [9], where a bank of adaptive observers, using only measurements and information from neighboring subsystems, are used to detect and isolate faults in interconnected subsystems. In [10], a FDI approach for distributed and heterogeneous networked systems is proposed, where each agent can detect and isolate both its own faults and the faults of its nearest neighbors.

In previous works [11], [12], some of the authors of this paper proposed a distributed fault diagnosis and isolation (FDI) approach for networked robots characterized by a continuous first order integrator dynamics, that allows each robot of the team to detect faults occurring on any other robot, even if not directly connected.

As agents are digitally controlled and explicit communication between agents happens at discrete time instants, such an approach has been extended to the framework of discrete-time systems, where each agent is characterized by a general discrete time linear model subject to a general state feedback input. In the designed strategy;

- each agent runs a distributed discrete-time local observer to estimate the overall state of the system. Based on this estimate, it is possible to compute the local control input;
- based on the same information needed by the local observer, each agent computes a residual vector relative to the other teammates;
- these residuals are designed in such a way that if a fault occurs on the corresponding agent, they overcome a proper designed threshold. In this way, a fault relative to an agent can be detected and isolate by all the team members;
- once faults are identified, the mission is rescheduled accordingly.

A rigorous analysis and conditions on the observer convergence and on the residual dynamics was carried out.

The dynamics of each agent in the team composed by N agents is assumed to be

$$\boldsymbol{x}_i(k+1) = \boldsymbol{A}\boldsymbol{x}_i(k) + \boldsymbol{B}\left(\boldsymbol{u}_i(k) + \boldsymbol{\phi}_i(k)\right) + \boldsymbol{\zeta}_i(k), \tag{1}$$

with, $k \in Z$, $i = 1, 2, \ldots, N$, and where $\boldsymbol{x}_i(k) \in \mathbb{R}^n$, $\boldsymbol{u}_i(k) \in \mathbb{R}^p$, are, respectively, the agent's state and control input, $\boldsymbol{\phi}_i \in \mathbb{R}^p$ is an actuator fault, that is zero in healthy conditions, and $\boldsymbol{\zeta}_i \in \mathbb{R}^n$ is the process noise. The local control input of the system in (1) would be

$${}^{i}\hat{\boldsymbol{u}}(k) = \boldsymbol{\Gamma}_i \boldsymbol{K}\,{}^{i}\hat{\boldsymbol{x}}(k) + \boldsymbol{u}_{ff}(k), \tag{2}$$

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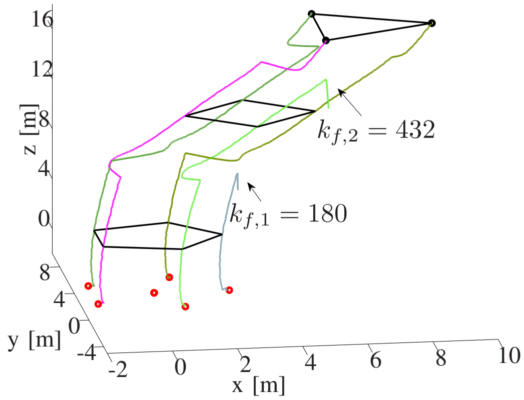

Fig. 1. The vehicle paths. Red bullets represent the starting points, black bullets the ending points and continuous lines the vehicle paths. The black pentagon represents the formation at $k = 120$, the black square at $k = 340$ and the black triangle at $k = 600$. $k_{f,1}$ and $k_{f,5}$ are the fault instants affecting vehicle 1 and 5, respectively.

where $\boldsymbol{\Gamma}_i$ is a selection matrix, ${}^i\hat{\boldsymbol{x}}(k)$ is the estimate of the state of the overall system made by i th agent, $\boldsymbol{K} \in \mathbb{R}^{pN \times Nn}$ is a positive definite constant gain matrix, and $\boldsymbol{u}_{ff}(k) \in \mathbb{R}^{pN \times Nn}$ represents a feed-forward term.
As an example, let us consider case of a team composed by 5 vehicles that has to move in the 3D space while keeping a regular formation. Each agent has a second order continuous dynamics:

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{I}_3 \otimes \boldsymbol{A}_c \boldsymbol{x}(t) + \boldsymbol{I}_3 \otimes \boldsymbol{B}_c \boldsymbol{u}(t) \tag{3}$$

where

$$\begin{aligned} \boldsymbol{x} &= \begin{bmatrix} x, & \dot{x}, & y, & \dot{y}, & z, & \dot{z} \end{bmatrix}^{\mathrm{T}}, \\ \boldsymbol{u} &= \begin{bmatrix} u_x, & u_y, & u_z \end{bmatrix}^{\mathrm{T}}, \\ \boldsymbol{A}_c &= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \quad \boldsymbol{B}_c = \begin{bmatrix} 0 \\ 1 \end{bmatrix}. \end{aligned} \tag{4}$$

If a digital controller is adopted to control the system (3), the input $\boldsymbol{u}(t)$ is updated at fixed sampling time T and maintained constant over the sampling interval by the D/A converter, i.e. $\boldsymbol{u}(kT + t) = \boldsymbol{u}(kT)\, \forall k \in Z,\ \text{for } 0 \leq t < T$.

Therefore, it makes sense to consider a discrete-time equivalent model of (3), namely

$$\boldsymbol{x}(k+1) = \boldsymbol{I}_3 \otimes \boldsymbol{A}_d \boldsymbol{x}(k) + \boldsymbol{I}_3 \otimes \boldsymbol{B}_d \boldsymbol{u}(k), \tag{5}$$

where kT has been replaced by k and

$$\begin{aligned} \boldsymbol{x}(k) &= \left[x(kT)\, \dot{x}(kT)\, y(kT)\, \dot{y}(kT)\, z(kT)\, \dot{z}(kT)\right]^{\mathrm{T}}, \\ \boldsymbol{u}(k) &= \left[u_x(kT)\, u_y(kT)\, u_z(kT)\right]^{\mathrm{T}}, \\ \boldsymbol{A}_d &= \exp(\boldsymbol{A}T) = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix},\ \boldsymbol{B}_d = \int_0^T \exp(\boldsymbol{A}(T-\tau))\boldsymbol{B}d\tau = \begin{bmatrix} T^2/2 \\ T \end{bmatrix}. \end{aligned} \tag{6}$$

During the mission, actuator faults occur on vehicles 1 and 5 at steps $k = 180$ and $k = 432$, respectively. In Fig. 1, the vehicle paths are depicted together with the instants at which the faults start affecting vehicle 1 and 5. With regards to the fault detection and isolation, Fig. 2 shows the residual norms (blue continuous lines) and the corresponding thresholds (dashed green line) relative to a healthy vehicle (namely vehicle 2 as an example) as calculated by the other ones ($\left\|{}^i\boldsymbol{r}_2\right\|$, $i = 1, 2, 3, 4$). As it can be seen, residual norms are at any instant below the corresponding thresholds and, thus, no fault is correctly detected. Figures 3 show how fault on vehicle 1 affects the residual components (relative to it) of all the teammates. In detail, the residual norms $\left\|{}^i\boldsymbol{r}_1\right\|$ $(i = 1, 2, 3, 4, 5)$ quickly overtake the thresholds few steps after the fault occurrence, making the faults correctly detected by the team. The same happens for vehicle 5.

References

[1] F. Bullo, J. Cortés, and S. Martínez, *Distributed Control of Robotic Networks*, ser. Applied Mathematics Series. Princeton University Press, 2009.
[2] R. Olfati-Saber, J. Fax, and R. Murray, "Consensus and cooperation in networked multi-agent systems," *Proceedings of the IEEE*, vol. 95, no. 1, pp. 215–233, Jan. 2007.

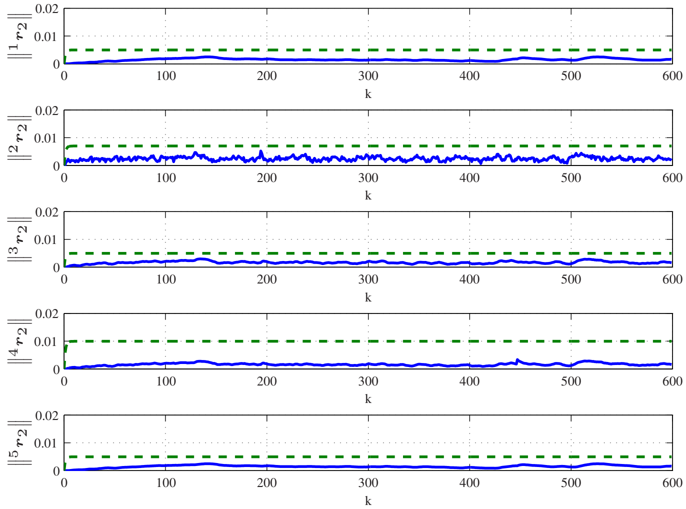

Fig. 2. Residuals $\|{}^{i}\boldsymbol{r}_2\|$ ($i = 1, 2, 3, 4, 5$) (solid blue lines) as calculated by vehicle i and relative to healthy faulty vehicle 2. Dashed green lines are the corresponding thresholds.

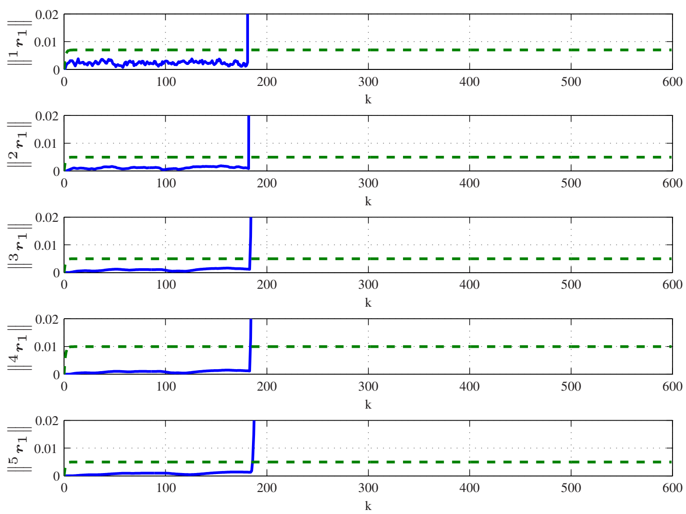

Fig. 3. Residuals $\|{}^{i}\boldsymbol{r}_1\|$ ($i = 1, 2, 3, 4, 5$) (solid blue lines) as calculated by vehicle i and relative to the faulty vehicle 1. Dashed green lines are the corresponding thresholds.

[3] W. Ren and R. Beard, *Distributed Consensus in Multi-vehicle Cooperative Control*, ser. Communications and Control Engineering. Berlin, G: Springer, 2008.

[4] C. Belta and V. Kumar, “Abstraction and control of groups of robots,” *IEEE Transactions on Robotics*, vol. 20, no. 5, pp. 865–875, 2004.

[5] G. Antonelli, F. Arrichiello, F. Caccavale, and A. Marino, “Decentralized time-varying formation control for multi-robot systems,” *The International Journal of Robotics Research*, in press, 2014.

[6] X. Zhang, M. Polycarpou, and T. Parisini, “A robust detection and isolation scheme for abrupt and incipient faults in nonlinear systems,” *IEEE Transactions on Automatic Control*, vol. 47, no. 4, pp. 576–593, 2002.

[7] F. Caccavale, A. Marino, G. Muscio, and F. Pierri, “Discrete-time framework for fault diagnosis in robotic manipulators,” *IEEE Transactions on Control Systems Technology*, vol. 21, no. 5, pp. 1858–1873, 2013.

[8] R. Ferrari, T. Parisini, and M. Polycarpou, “Distributed fault diagnosis with overlapping decomposition: an adaptive approximantion approach,” *IEEE Trans. on Automatic Control*, vol. 54, no. 4, pp. 794–799, 2009.

[9] X. Zhang and Q. Zhang, “Distributed fault diagnosis in a class of interconnected nonlinear uncertain systems,” *International Journal of Control*, vol. 85, no. 11, pp. 1644–1662, 2012.

[10] M. Davoodi, K. Khorasani, H. Talebi, and H. Momeni, “Distributed fault detection and isolation filter design for a network of heterogeneous multiagent systems,” *IEEE Transactions on Control Systems Technology*, vol. 22, no. 3, pp. 1061–1069, 2014.

[11] F. Arrichiello, A. Marino, and F. Pierri, “A decentralized observer for a general class of lipschitz systems,” in *Proceedings 2013 Int. Conf. on Advanced Robotics*, Montevideo, Uruguay, November 2013.

[12] ——, “Observer-based decentralized fault detection and isolation strategy for networked multirobot systems,” *IEEE Transactions on Control Systems Technology*, vol. PP, no. 99, 2015.